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Some Problems of the Static Direct Transformation

ZHANG Zhongren

Introduction

The Transformation Problem is a problem concerning the transformation from value to the production-price, and the function of its mathematics model is to establish a link between the different two systems. All these can be summed up as follows: supposing that the value system is known[1], and starting from this known value system, calculate the unknown production pricing system; that is, to find the deviation parameter of the production-price from value.

The difficulty of the Transformation Problem lies in whether it is possible to satisfy the following twp restrictions after transformation, namely: total average profit equals to total surplus value, and total of production-price equals to total value. Regarding these two restrictions, there have not been established any adequate results which satisfy them completely.

Mach controversy has been arising on the transformation problem since Karl Marx's *Capital* Vol. Ⅲ was published. Particularly since L. von Bortkiewicz proposed in 1907 a calculus method of transformation, we have observed two worldwide debates in more than 90 years (especially the debate in the early 1970s between P. A. Samuelson and M. Morishima). Even now, it remains an unsettled question. But I have based on the coefficient method proposed by L. von Bortkiewicz [1907], by absorbing merits of methods proposed by F. Seton [1957] and P. A. Samuelson [1957], set up a generic transformation model for Marx's *"two- invariance"*[2] subject can be simultaneously tenable after transformation, which is consistent with Karl Marx's original intent, and with this as a base, to set up the so-called inverse transformation model from the production-price to value, as an inverse function of the former model, in 2000 year[3]. But what is mentioned above only points out and proves the existence of unique solution of the model. Actually, the transformation model not only has unique solution, and also the solution is positive.

In fact, if it is impossible to ensure the model has positive solution, it will be less significant in economics.

The object of this paper is to give stringent mathematics proof of existence and uniqueness and positive of the solutions in the transformation model, and at the same time, has solved the subject concerning *unit* of transformation.

The following is a brief explanation of the symbols used in this paper:

1. c_i, v_i, m_i and w_i represent respectively the constant capital, variable capital, surplus value and total value in the i^{th} department. (c_i+v_i) is the total capital of the i^{th} department, and it is also called cost, represented with h_i; needless to say, $h_i = c_i+v_i$.

TABLE 1

Input-Output Table of n Department (Value System)

departments	1	2	…	n	variable capital	surplus value	total value
1	c_{11}	c_{12}	…	c_{1n}	v_1	m_1	w_1
2	c_{21}	c_{22}	…	c_{2n}	v_2	m_2	w_2
…	…	…	…	…	…	…	…
n	c_{n1}	c_{n2}	…	c_{nn}	v_n	m_n	w_n
final use	y_1	y_2	…	y_n			
gross output	w_1	w_2	…	w_n			

As shown in Table 1, the supply-demand equilibrium relation during that year in $n(n \geqq 2)$ departments can be represented in the following formula (y_i is the final use):

$$\sum_{j=1}^{n} c_{ij}+v_i+m_i = \sum_{h=1}^{n} c_{hi}+y_i = w_i \qquad (i = 1, 2, \cdots, n) \qquad (1)$$

2. $e(= m_i/v_i\,; i = 1, 2, \cdots, n)$ represents surplus value rate.

3. H_i (in $= C_i+V_i$, the capital letter C_i, V_i shows the constant capital and variable capital in the i^{th} department, under the production-price) represents the cost under the production-price in the i^{th} department.

4. r is average profit rate, $S_i\,(= rH_i = r[C_i+V_i]$ is average profit in the i^{th} department.

5. P_i is the total the production-price in the i^{th} department; obviously, $P_i = H_i+S_i = (1+r)H_i$. Formula (1) can be called value system. In the same way, supply-demand equilibrium relation during that year in the production-price can be represented in the following formula (2):

$$\sum_{j=1}^{n} C_{ij}+V_i+S_i = (1+r)(\sum_{j=1}^{n} C_{ij}+V_i) = \sum_{h=1}^{n} C_{hi}+Y_i = P_i\,(i = 1, 2, \cdots, n) \qquad (2)$$

In Formula (2), the most important part is the following (3):

$$(1+r)(\sum_{j=1}^{n} C_{ij}+V_i) = P_i\,(i = 1, 2, \cdots, n) \qquad (3)$$

Mathematics models are always based on certain postulates as their premises, so is the Transformation mathematics model. Here the following points are the premises: surplus value rate of all departments is the same; technology remains unchanged; for all kinds of capital, the yearly circulation rate is 1.

1. Mathematics Proof for Existence and Uniqueness and Positive of Solutions in the Transformation Model

We have overcame that there exist two serious errors in the past research for the study of the Marx's transformation problem until now[4], and have got a transformation model[5]. That is as follows:

$$\left.\begin{array}{l}(1+r)(\sum_{j=1}^{n} c_{ij}+x_j+v_i y) = w_i x_i \ (i=1, 2, \cdots, n)\\ \sum_{i=1}^{n} w_i x_i = \sum_{i=1}^{n} w_i \\ r = \sum_{i=1}^{n} m_i / \sum_{i=1}^{n}(c_i+v_i)\end{array}\right\} \quad (4)$$

The significance of the transformation model (4) mentioned above is to satisfy the ***"two-invariance"***[6] subject. As the $(n+1)^{\text{th}}$ equation can ensure total quantity of the production-price equal to total magnitude of value, with the precedent n equations added together, we have $(1+r)\sum H_i = \sum w_i$. Therefore,

$$\sum S_i = r\sum H_i = \frac{r}{1+r}\sum w_i = \frac{\dfrac{\sum m_i}{\sum h_i}}{1+\dfrac{\sum m_i}{\sum h_i}}\sum w_i = \sum m_i \quad (5)$$

Namely, total quantity of average profit equals total magnitude of surplus-value. (Refer to the definition of average profit and the proof of Theorem 2 hereinafter.)

But what is mentioned above only points out and proves the existence of unique solution of the model. Actually, the model not only has unique solution, and also the solution is positive. In fact, if it is impossible to ensure the model has positive solution, it will be less significant in economics. We shall prove this conclusion in the following description.

The last equation of (4) is the definition formula of r. Therefore, transformation model (4) contains only $(n+1)$ unknown quantities and $(n+1)$ equations. Let

$$\mathbf{A} = \begin{bmatrix}(1+r^*)\mathbf{C}-\overline{\mathbf{W}} & (1+r^*)\mathbf{V}\\ \mathbf{W}' & 0\end{bmatrix}, \mathbf{Z} = \begin{bmatrix}\mathbf{X}\\ y\end{bmatrix}, \mathbf{M} = \begin{bmatrix}\mathbf{0}\\ \omega\end{bmatrix}$$

Here $\omega = w_1+w_2+\cdots+w_n$, $\mathbf{W}' = (w_1, w_2, \cdots w_n)$, $\mathbf{X} = (x_1, x_2, \cdots x_n)'$, $\mathbf{V} = (v_1, v_2, \cdots, v_n)'$, $\overline{\mathbf{W}} = diag(w_1, w_2, \cdots, w_n)$. Then (4) set can be transformed into the following matrix:

$$\begin{bmatrix}(1+r^*)\mathbf{C}-\overline{\mathbf{W}} & (1+r^*)\mathbf{V}\\ \mathbf{W}' & 0\end{bmatrix}\begin{bmatrix}\mathbf{X}\\ y\end{bmatrix} = \begin{bmatrix}\mathbf{0}\\ \omega\end{bmatrix} \quad (6)$$

or

$$\mathbf{AZ} = \mathbf{M} \quad (7)$$

In the following description, we shall prove the existence of unique solution to (7) and the solution is positive[7]. For this, we shall first prove two lemmas

Lemma 1. *In the following determinant* $D < 0 (n \geqq 2)$,

$$D = \begin{vmatrix} a_1 & b_{12} & \cdots & b_{1,n-1} & d_1 \\ b_{21} & a_2 & \cdots & b_{2,n-1} & d_2 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ b_{n-1,1} & b_{n-1,2} & \cdots & a_{n-1} & d_{n-1} \\ c_1 & c_2 & \cdots & c_{n-1} & M \end{vmatrix}$$

Where ,

$a_i > 0, d_i < 0$ $(i = 1, 2, \cdots, n-1)$, $c_j < 0$ $(j = 1, 2, \cdots, n-1)$,

$b_{ij} \leq 0$ $(i, j = 1, 2, \cdots, n-1 ; i \neq j)$, $M \leq 0$.

Proof. Mathematic induction is used in the proof.

(1) When $n = 2$, as $a_1 > 0$, $M \leq 0$, so $a_1 M \leq 0$, and as $c_1, d_1 < 0$, we know $c_1 d_1 > 0$, then

$$D = \begin{vmatrix} a_1 & d_1 \\ c_1 & M \end{vmatrix} = a_1 M - c_1 d_1 < 0 \tag{8}$$

(2) Suppose that in the case of , the subject is true, and in the case of , because $a_1 > 0$, by adding several times a_1 to elements after a_1 on the first row to make them equal to 0, then a new determinant $\overline{D}$ equivalent to determinant D is obtained. In this new determinant,

$$b'_{ij} = b_{ij} - \frac{b_{i1} b_{1j}}{a_1} \leq 0 \ (i, j = 2, \cdots, n-1 ; i \neq j) \tag{9}$$

this is because $b_{ij} \leq 0$ $(i, j = 1, 2, \cdots n-1 ; i \neq j)$, and $a_1 > 0$, $b_{i1} b_{1j} \geq 0$. For the same reason,

$$c'_j = c_j - \frac{c_1 b_{1j}}{a_1} < 0 \ (j = 2, \cdots, n-1) ; d'_i = d_i - \frac{b_{i1} d_1}{a_1} < 0 \ (i = 2, \cdots, n-1) \text{ and}$$

$$M' = M - \frac{c_1 d_1}{a_1} < 0 \tag{10}$$

Therefore, the complementary minor of a_1 is D_{n-1} in the new determinant $\overline{D}$ satisfies the condition mentioned in this lemma, in accordance with the postulation in the case of $n-1$, $D_{n-1} < 0$, so that

$$D = \overline{D} = a_1 D_{n-1} < 0 \tag{11}$$

The proof is completed.

Corollary 1. *For the coefficient matrix* $\mathbf{A}$ *of equation set* (7), *there exists*

$$\det(-\mathbf{A}) < 0 \tag{12}$$

For this corollary, we should note that as $v_i > 0$, *elements of* $(1+r^*)\mathbf{C} - \overline{\mathbf{W}}$ *on the main diagonal line, namely,* $(1+r^*) c_{ij} - w_i < 0$ *is enough.*

Corollary 2. *If* $M < 0$, *then when* $c_i \leq 0$ $(i = 1, 2, \cdots, n-1)$, *then* $D < 0$ *also exists.*

Lemma 2. *Determinant* $\det(\overline{\mathbf{W}}-(1+r^*)\mathbf{C}) > 0$.

Proof. Let $g_{ij} = c_{ij}/w_i$, then $\overline{\mathbf{W}}-(1+r^*)\mathbf{C}$ can be transformed into $\overline{\mathbf{W}}[\mathbf{I}-(1+r^*)\mathbf{G}]$, and as the sum of elements in every row of matrix $\mathbf{I}-(1+r^*)\mathbf{G}$ are all less than 1, satisfying the so-called R.Solow condition for row, so that $\det(\mathbf{I}-(1+r^*)\mathbf{G}) > 0$, and obviously $\det(\overline{\mathbf{W}}) > 0$, therefore,

$$\det(\overline{\mathbf{W}}-(1+r^*)\mathbf{C}) = \det(\overline{\mathbf{W}}[\mathbf{I}-(1+r^*)\mathbf{G}]) = \det(\overline{\mathbf{W}})\det([\mathbf{I}-(1+r^*)\mathbf{G}]) > 0$$

The proof is completed.

Theorem 1. *Transformation model* $\mathbf{AZ} = \mathbf{M}$ *has sole solution, and the solution is positive.*

Proof. In order to make use of the conclusion from Corollary 1, we first construct an equation set equivalent to $\mathbf{AZ} = \mathbf{M}$:

$$-\mathbf{AZ} = -\mathbf{M} \tag{13}$$

From Corollary 1 of Lemma 1, we know that the coefficient matrix of (13) has a determinant $\det(-\mathbf{A}) < 0$, so that (13) has solution and the solution is sole. Transform the $j^{\text{th}}(1 \leqq j \leqq n)$ row into $-\mathbf{M}$ to form a determinant represented by B_j, then ω is on the $(n+1)^{th}$ row and j^{th} column of B_j, and the complementary minor of ω is represented as B_j^*; and change the j^{th} row of B_j^* respectively into n^{th} row, to form a determinant represented as B_j^{**}, then

$$B_j^{**} = \begin{vmatrix} w_1-(1+r^*)c_{11} & \cdots & -(1+r^*)c_{1,j-1} & -(1+r^*)c_{1,j+1} & \cdots & -(1+r^*)c_{1n} & -v_1 \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -(1+r^*)c_{j-1,1} & \cdots & w_{j-1}-(1+r^*)c_{j-1,j-1} & -(1+r^*)c_{j-1,j+1} & \cdots & -(1+r^*)c_{j-1,n} & -v_{j-1} \\ -(1+r^*)c_{j+1,1} & \cdots & -(1+r^*)c_{j+1,j-1} & w_{j+1}-(1+r^*)c_{j+1,j-1} & \cdots & -(1+r^*)c_{j+1,n} & -v_{j+1} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ -(1+r^*)c_{n,1} & \cdots & -(1+r^*)c_{n,j-1} & -(1+r^*)c_{n,j+1} & \cdots & w_n-(1+r^*)c_{nn} & -v_n \\ -(1+r^*)c_{j,1} & \cdots & -(1+r^*)c_{j,j-1} & -(1+r^*)c_{j,j+1} & \cdots & -(1+r^*)c_{j,n} & -v_j \end{vmatrix}$$

According to Corollary 2 of Lemma 1, we know $B_j^{**} < 0$, therefore, by Gramer rule,

$$x_i = \frac{B_j}{\det(-\mathbf{A})} = \frac{(-1)^{n+1+j}(-\omega)B_j^*}{\det(-\mathbf{A})} = \frac{(-1)^{n+2+j}(-1)^{n-j}\omega B_j^{**}}{\det(-\mathbf{A})} = \frac{\omega B_j^{**}}{\det(-\mathbf{A})} > 0 (i = 1, 2, \cdots, n)$$

By the same reason, change the $(n+1)^{th}$ column of $\det(-\mathbf{A})$ into $-\mathbf{M}$ to form a determinant expressed as B_{n+1}, then

$$y = \frac{B_{n+1}}{\det(-\mathbf{A})} = \frac{-\omega det(\overline{\mathbf{W}}-(1+r^*)\mathbf{C})}{\det(-\mathbf{A})} > 0$$

Note that here the conclusion of Lemma 2 is used. The proof is completed.

In this way, the generic solution of transformation model (6) can be expressed in a form of matrix, namely,

$$\begin{bmatrix} \mathbf{X} \\ y \end{bmatrix} = \begin{bmatrix} (1+r^*)\mathbf{C}_{n\times n} - \overline{\mathbf{W}}_{n\times n} & (1+r^*)\mathbf{V}_{n\times 1} \\ \mathbf{W}'_{1\times n} & 0 \end{bmatrix}^{-1} \begin{bmatrix} 0_{n\times 1} \\ \omega \end{bmatrix} \tag{14}$$

The core of the nearly 100 years old debate on transformation problem is merely whether the "two equivalences" can hold after transformation. So long as the "***two－invariance***" problem is proved, we can properly say that the transformation problem is ended. The transformation model (4) and theorem 1 accomplish such a mission.

2. The Third Erroneous Domain is the Unit Problem in the Study of Transformation Problem

We have pointed out the two errors of cognition, in the study of transformation problem, which are the fundamental factor that disturbed people who attempted to solve the problem for nearly one hundred years. But actually, there is another error, unit problem; namely, the production-price being seen as price and thus transformation is seen as an intermediate link between transformations from value to price.

Originally, there was no link between transformation problem and price problem. Actually, the production-price is nothing but a state of price transformation in the exchange process when redistribution occurs in surplus-value as a result of competition, whereas price is manifestation of value in currency, which forms in another process irrelevant to price problem and is completely competent for solving transformation problem. Therefore, expounding the two errors as mentioned above is enough for deriving transformation calculus model.

But many researchers of transformation problem erroneously deem that Karl Marx's value theory and price theory are disconnected and try to seek out the relationship between value and price in the transformation process, which leads the study of transformation problem to a wrong road. As a matter of fact, Karl Marx made detailed description on the formation process of price in section 3, chapter 1 of *Capital* Vol. Ⅰ, but he did not make it into a mathematics model. This is another process different from transformation process.

Just as Karl Marx said in the preface of *Capital* (first edition of Vol. Ⅰ), "With the exception of the section on value-form, therefore, this volume cannot stand accused on the score of difficulty, "[8] problems occur in the understanding of this portion about value form. Of course, I do not mean any error in understanding, rather, I mean that people have not started from value form to derive Marx's price model; for this, many Western scholars have always deemed that there is no relationship between value and price in Marx's theory, and then they deem that value is superfluous.

Let us first look at the equality below:

$$20 \textit{ yards of linen} = 1 \textit{ piece of coat} \tag{15}$$

In sense of economics, the value manifestation of linen is relative value, or in other words, it is in a form of relative value, and the coat plays a role of equivalent, or in other words, it is in a form of equivalence. But in sense of mathematics, the both sides of (15) as use value, not only have different units but also unequal in quantity, so, how can this equality hold? Karl Marx explained like these: "The equation, 20 yards of linen=1 coat, or 20 yards of linen are worth one coat, implies that the same quantity of value-substance (congealed labour) is embodied in both; that the two commodities have each cost the same amount of labour of the same quantity of labour-time."[9] In short, equality (15) holds because the both sides contain commensurate labor time.

Now let us write quality (15) in another form—to derive the labor time consumed in the two commodities so as to make it hold in sense of mathematics. Let l_1 express the labor-time socially necessary for producing 1 yard of linen, its unit is hour/yard; and let l_2 express the labor-time socially necessary for producing 1 piece of coat, its unit is hour/piece. If $l_1 = 3$ hour/yard (in other words, the value of 1 yard linen is 3 hours, or, the unit value of linen is 3 hours), then obviously $l_2 = 60$ hours/piece. According to Marx's expression, (15) should be rewritten as

$$20\ yards \times l_1\ hour/yard = 1\ piece \times l_2\ hour/piece \tag{16}$$

or

$$20\ yards \times 3\ hours/yard = 1\ piece \times 60\ hours/piece \tag{17}$$

(17) reflects equivalent exchange between two commodities in total value, and the result is

$$60\ hours = 60\ hours$$

In the above equation, unit value of linen is obviously $l_1 = 3$ hours/yard, but as linen cannot express value of its own, in other words, value of linen can be expressed relatively, that is, expressed by another commodity, therefore, value of linen can be expressed only by coat; and for coat, the same problem exists. Therefore, we cannot know whether l_1 equals to 3 hours/yard, rather, we can only derive

$$1\ yard \times l_1\ hour/yard = 1/20\ piece \times l_2\ hour/piece \tag{18}$$

through equation (15).

In *Capital*, the generic expression of equation (15) is x quantity of commodity $A = y$ quantity of commodity B. It can be re-written as

$$q_i(d_i) \times l_i(h/d_i) = q_j(d_j) \times l_j(h/d_j) \tag{19}$$

just as we did with equation (17).

Here q_k represents the quantity of k^{th} commodity, its unit is d_k, and h is used to represent time, l_k represents the labor-time socially necessary, its unit is h/d_k; in other words, l_k represents that one unit value (d_k) of the k^{th} commodity is l_k hour[10].

In the case of currency, the expression is the same, except that it should be considered that 20 yards of linen = 2 ounces of gold. Let us use l_0 to express the labor-time socially necessary for producing 1 ounce of gold, then its unit is hour/ounce, and the exchange equation is

$$20\ (yards) \times l_1\ (hour/yard) = 2\ (ounces) \times l_0\ (hour/ounce) \tag{20}$$

Karl Marx's currency form is expressed like these:

$$\left.\begin{array}{l} 20\ \textit{yards of linen} = \\ 1\ \textit{piece of coat} = \\ 10\ \textit{pounds of tea} = \\ 10\ \textit{pounds of coffee} = \\ 1\ \textit{quart of wheat} = \\ 1/2\ \textit{tone of iron} = \\ x\ \textit{quantity of commodity}\ A = \end{array}\right\}\ 2\ \textit{ounces of gold} \qquad (21)$$

Of them, the last expression x quantity of commodity $A = 2$ ounces of gold actually generalizes the subject. In chapter 3 of *Capital* Vol. I, it is further generalized as x quantity of commodity $A = y$ quantity of currency. This equation can be expressed in our method like this:

$$q_k(d_k)\times l_k(h/d_k) = q_0(d_0)\times l_0(h/d_0) \qquad (k = 1, 2, \cdots, n) \qquad (22)$$

Here, q_0 represents the quantity of gold, its unit is d_0 ounce. But equation (22) remains in generic value form, because the result of exchange is labor time. After currency forms, the result of exchange should be currency. This change comes from money to itself as its own equivalent[11] Therefore, in quantity, the quantity of unit value l_k is transformed into l_k/l_0. In sense of unit, unit value l_k is transformed from $\frac{h/d_k}{h/d_0}$ into d_0/d_k, while in sense of economics, the unit value l_k expressed by currency is transformed into price l_k/l_0. In mathematics, such a change is reflected on (22), but it is only simple equivalent exchange. Embodied on (20), such a change becomes

$$20\ \textit{years}\times l_1/l_0(\textit{ounce/yard}) = 2\ \textit{ounces} \rightarrow l_1/l_0(\textit{ounce/yard}) = \frac{1}{10}(\textit{ounce/yard}) \quad (23)$$

Here, l_1/l_0 is price of linen, but as l_0 and l_1 cannot express themselves, it is expressed through the proportion between the quantity of gold exchanged from 20 yards of linen and the quantity of 20 yards of linen. (23) reflects the relation between invisible value and visible price.

After further generalization, we have

$$\frac{l_k}{l_0}(d_0/d_k) = \frac{q_0}{q_k}(d_0/d_k) \qquad (k = 1, 2, \cdots, n) \qquad (24)$$

Here, $1/l_0$ is value-price exchange coefficient, although we have no way to calculate it, by its theoretic significance, it is enough for us to clear up obstacle on the way to solving transformation problem. In particular, in study of transformation problem, value is postulated to be known number, transformation problem can be mathematically summed up as a problem of derivation from known value system to unknown price system. Therefore, in study of transformation problem, l_1/l_0 can be derived from known value system.

In the following description, we shall popularize value-price relation to the whole value system and further solve the transformation problem under different units.

We use l_i to express the labor-time socially necessary for producing i^{th} commodity, the unit of time is expressed in hour, and the unit of use value of the i^{th} commodity is expressed as , and wage rate is expressed as , and and represent respectively output of real thing, correspondent to w_i and c_{ij}. And L_i represents the quantity of used labor correspondent to v_i, then (1) can change to

$$\sum_{j=1}^{n} q_{ij}^{c} l_j + (1+e)\delta_i L_i = q_i l_i \tag{25}$$

If the f^{th} commodity is gold (currency), its unit is d_f (ounce), its the labor-time socially necessary is l_f, then dividing the two sides of (25) with l_f,, we have

$$\sum_{j=1}^{n} q_{ij}^{c} \frac{l_i}{l_f} + (1+e) L_i \frac{\delta_i}{l_f} = q_i \frac{l_i}{l_f} \tag{26}$$

In this way, the price of i^{th} commodity is l_i/l_f, its unit is $(h/d_i)/(h/d_f) = d_f/d_i$. In other words, d_i unit of i^{th} commodity is worth l_i/l_f ounces of gold. (26) is the further general form of (24). Here, $1/l_f$ is value-price exchange coefficient[12]. In numerical value, price is only $1/l_f$ time the value.

In the same way, transformation model (4) can be changed into

$$\left.\begin{aligned} &(1+r)\Big[\sum_{j=1}^{n} q_{ij}^{c}\Big(\frac{l_j}{l_f x_f} x_j\Big) + L_i\Big(\frac{\delta_i}{l_f x_f} y\Big)\Big] = q_i\Big(\frac{l_i}{l_f x_f} x_i\Big) \quad (i = 1, 2, \cdots, n) \\ &\sum_{i=1}^{n} q_i\Big(\frac{l_i}{l_f x_f} x_i\Big) = \sum_{i=1}^{n} q_i \frac{l_i}{l_f x_f} \end{aligned}\right\} \tag{27}$$

Note that in (27), the definition equation of average profit rate is omitted. In essence, transformation is from the labor-time socially necessary l_i to $l_i x_i$, while the labor-time socially necessary of currency as a commodity transforms from l_f to $l_f x_f$. Equation (27) is just a transform model in manifestation of currency. The solution of (27) is the same as that of (4). Price, as manifestation of the production-price in currency form, is $1/x_f$ time of price, as manifestation of value in currency form; $1/x_f$ embodies the change of value in currency form, but the "a change in the value of gold does not, in any way, affect its function of gold as standard of standard price. … a change in the value of gold does not interfere with its function as a measure of value."[13]

As for the respect that the production-price is a converted form of value[14], seemingly, there is not disagreement in China economics circle. But in oversea academies, many people treat the production-price as price, and then transformation is treated as an intermediate link in the transformation between value and price. This is actually a serious interpretation of Marxist theory about the production-price.

It is improper to say that Bortkiewicz left unconsciously a "pitfall" in his research in 1907. As a matter of fact, Bortkiewicz did not intend to treat the production-price as price. Although he often abbreviated the production-price to price, he did not mix up the production-price with price. Actually, he was not involved in the subject of relation between value and price. But as it is well known, Bortkiewicz's 1907 model has four unknown numbers and only three equations. To make this model solvable, he managed to cut down on unknown numbers. He deemed that if price (referred to as the production-price) and value have the same unit, then it is proper to think that among the three sectors, there must be one sector producing a commodity that can be the unit of price and value. Supposedly, this commodity is gold, then, the fourth unknown number $z = 1$[15]. Here, Bortkiewicz said merely how to express value and the production-price with currency, rather to mix up the production-price and price. But in the analysis of P. M. Sweezy, in order to expound the meaning mentioned above, he said that the number of unit labor time required for producing one unit of currency can link the two calculus systems. What he said is actually exchange coefficient between value and price. The so-called "two calculus systems" was referred to as value and price, rather than value and the production-price[16]. But this "two calculus systems" was misinterpreted as value and the production-price. This misinterpretation was fully admitted as right by H. D. Dickinson in 1956 in his short paper, making comment on R. L. Meek's thesis, saying that value is measured by quantity of labor, price is measured by quantity of currency (here, if price is referred to as the production-price, it is wrong, but obviously, Dickson did so), then value in 3 different sectors is transformed respectively to price (referred to as the production-price) with multipliers x, y and z. These multipliers correspond to the factors linking labor time and currency, namely, transformation process is seen as formation process of price. This is a great error. Strangely, the Dickson's error has not met with sharp criticism. This indicates that such a chaotic cognition is quite universal.

On the relation between the production-price and value, only K. May correctly recognized that the production-price is a form of value. But her correct conclusion was retorted as "detached with the production-price theory", and compared with P. M. Sweezy and J. Wenternitz's solutions, "instead, it lags behind." Just in this way, the production-price mixes up with price, forming a serious erroneous domain in transformation study. "A leaf before the eye shuts out Mount Tai," this third erroneous domain straps study of transformation overseas. Even the NI (New Interpretation) or TSS (Temporal Single System) in the ascendant all have their focus on nothing but manifestation of labor in form of currency (MEI), although they each have their own merit in their concrete methods. These researchers are like "climbing a tree to catch fish", and they are increasingly far from the original intent of Marx, compared with the two worldwide debates.

In summary, the production-price is a special form of value, and price is a manifestation of value in the form of currency. Price is a currency form of value, and the process of its formation is merely a process in which a special commodity, (for example, gold) is separated to play the role of currency. After profit-averaging process is completed, price is no longer currency manifestation of value, but currency manifestation of the production-price. Transformation problem is originally irrelevant to price problem, because the process of price formation and the process of transformation are starkly different processes, but these two processes can be overlapped, in other words transformation process can be described either with labor time as unit (as Marx originally intended), or with currency as unit, while the result of transformation remains unchanged.

3. Other Three Propositions

By the summarize of M. Morishima, the following five conclusions are particularly important for the transformation problem.

(1) "the sum of the price of production of all commodities product in society — the totality of all branches of production — is equal to the sum of their values."[17)]

(2) "It remains true, nevertheless, that the cost-price of a commodity is smaller than its value."[18)]

(3) "Surplus-value and profit are identical from the standpoint of their mass."[19)]

(4) "Aside from possible differences in periods of turnover, the price of production of the commodities would then equal their value only in spheres, in which the composition [of capital] would happen to be [the same]."[20)]

(5) "The value of the commodities produced by capital [of higher value composition] would, therefore, be smaller than their price of production, the price of production of the commodities [produced by capital of lower composition] smaller than their value."[21)]

Those conclusions also are five propositions. Namely we must solve five propositions for the transformation problem. But proposition 1 and 2 also is aforesaid "*two- invariance*" (Namely, after transformation total the production-price equals total value, and total average profit equals total surplus-value. Those has been proved by the transformation model (4) in Z. ZHANG[2000]).

The proof of the proposition 3 is simple. In the transformation model (4), as $r > 0$ so

$$(\sum_{j=1}^{n} c_{ij}x_j + v_i y) < w_i x_i \qquad (i = 1, 2, \cdots, n) \tag{28}$$

Namely, The proof of the proposition 3 is completed.

We don't know whether the proposition 4 is tenable or not. First of all, we try to expand the concept of the composition of capital, for solve this proposition.

Marx defines the concept of the composition of capital of all departments as c_i, and had extracted any interdepartments exchange relations for constant capital, in *capital*. But, because we must take transformation of cost-price into production price[22)], may define $c_i = \sum_{i=1}^{n} c_{ij}$. It means to expand the concept of the composition of average capital

As the definition of the composition of capital, $k_i = c_i/v_i$ is the composition of capital of the i^{th} department, $k = \sum c_i / \sum v_i$ is the composition of average social capital. If the composition of capital of the j^{th} department is the same as the composition of average social capital, then imply $c_i/v_i = k_i = k = \sum c_i / \sum v_i$, that is, $(c_i, v_i) = (\sum c_i, \sum v_i)$.

Because we must take many changes by transformation of cost-price into production price, the composition of capital of the i^{th} department may bee expanded to $(c_{i1}, c_{i2}, \cdots, c_{in}, v_i)$ from (c_i, v_i). Here $\sum_{j=1}^{n} c_{ij} = c_i$, $c_{ij} \geq 0$, but $\sum_{j=1}^{n} c_{ij} = c_i > 0$. If $(c_{i1}, c_{i2}, \cdots, c_{in}, v_i) = (c_{h1}, c_{h2}, \cdots, c_{hn}, v_h)$, then $(\sum_{j=1}^{n} c_{ij}, \sum v_i) = (\sum_{j=1}^{n} c_{hj}, \sum v_h)$. But, generally the contrary is not tenable.

Definition1 if capital of the i^{th} department can satisfy

$$(c_{i1}, c_{i2}, \cdots, c_{in}, v_i) = q(\sum_{j=1}^{n} c_{j1}, \sum_{j=1}^{n} c_{j2}, \cdots, \sum_{j=1}^{n} c_{jn}, \sum_{j=1}^{n} v_j)$$

then the composition of capital is called *perfect composition of average capital*, and shows to $(c_{i1}^*, c_{i2}^*, \cdots, c_{in}^*, v_i^*)$.

But a point are emphasized here: as the composition of capital happens occasionally, perfect composition of average capital happens occasionally too.

Theorem2 the production-price is equal to the value, if perfect composition of average capital in the department.

Proof. Suppose capital of the i^{th} department is perfect composition of average capital, if we only can prove $x_i = 1$, then can think this proof is proved.

As the definition of perfect composition of average capital,

$$w_i^* = \frac{1}{q}\sum_{i=1}^{n} w_i \tag{29}$$

Sum up $1^{\text{th}} \sim n^{\text{th}}$ equations in the transformation model (4), that is,

$$(1+r)\left(\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij}^* x_j + \sum_{i=1}^{n} yv_i\right) = \sum_{i=1}^{n} w_i x_i \tag{30}$$

as $(n+1)^{\text{th}}$ equation in the transformation model (4), that is,

$$\sum_{i=1}^{n} w_i x_i = \sum_{i=1}^{n} w_i \tag{31}$$

so

$$w_i^* x_i = (1+r)\left(\sum_{j=1}^{n} c_{ij}^* x_j + yv_i^*\right) = \frac{1+r}{q}\left(\sum_{i=1}^{n}\sum_{j=1}^{n} c_{ij} x_j + \sum_{i=1}^{n} yv_i\right) = \frac{1}{q}\sum_{i=1}^{n} w_i = w_i^* \tag{32}$$

namely $x_i = 1$。

We can not prove yet that theorem2 is necessary and sufficient condition, only can prove "if the production-price is equal to the value in the department, then composition of capital of the department as composition of average capital to be the same". When the production-price is equal to their value in i^{th} department, that is

$$(1+r)\left(\sum_{j=1}^{n} c_{ij} x_j + v_i y\right) = w_i, \text{ so } m_i = S_i \text{ namely}$$

$$ev_i = \frac{r}{1+r} w_i, \text{ so }, v_i = \frac{r}{e}\cdot\frac{1}{1+r} w_i, \; c_i = \left(1-\frac{r}{e}\right)\frac{1}{1+r} w_i$$

Therefore, capital of the i^{th} department is

$$\frac{c_i}{v_i} = \frac{\left(1-\frac{r}{e}\right)\frac{1}{1+r} w_i}{\frac{r}{e}\cdot\frac{1}{1+r} w_i} = \frac{e-r}{r} = \frac{e-\sum_{i=1}^{n} ev_i / \sum_{i=1}^{n}(c_i+v_i)}{\sum_{i=1}^{n} ev_i / \sum_{i=1}^{n}(c_1+v_1)} = \frac{\sum_{i=1}^{n} c_i}{\sum_{i=1}^{n} v_i} \tag{33}$$

Namely composition of capital of i^{th} department as composition of average capital to be the same. But we don't know when the production-price equal the value, whether perfect composition of average capital is tenable or not, in the department.

Let $\Lambda = \{\alpha_i | \alpha_i = (c_{i1}, c_{i2}, \cdots, c_{in}, v_i), (i = 1, 2, \cdots, n); c_{ij} \geq 0, \sum_{j=1}^{n} c_{ij} > 0, v_i > 0\}$,
$\Gamma = \{\beta_i | \beta_i = (C_{i1}, C_{i2}, \cdots, C_{in}, V_i), (i = 1, 2, \cdots, n); C_{ij} \geq 0, \sum_{j=1}^{n} C_{ij} > 0, V_i > 0\}$,

then the transformation model (4) means a mapping(one to one function) from Λ to Γ. We show this mapping to $\beta = f(\alpha)$. Let $\overline{\mathbf{Z}}' = (\overline{x}_1, \overline{x}_2, \cdots, \overline{x}_n, \overline{y})$, $\mathbf{I}_0' = (1, 1, \cdots, 1)'_{1\times(n+1)}$, then, economic phenomena of the production-price is equal to the value means a plane through the points of $\overline{\mathbf{Z}}$ and $\mathbf{I}_0$, in mathematics[23], and $\alpha_i = (c_{i1}, c_{i2}, \cdots, c_{in}, v_i)$ is the normal of the plane. Because theorem2 is not necessary and sufficient condition, maybe the plane is not only one. Namely, there are some α by any possibility, and α is perpendicular to vector $\overrightarrow{\overline{\mathbf{Z}}\mathbf{I}_0}$. We show the plane (those α are located a plane) to Ψ, and clearly Ψ is perpendicular to vector $\overrightarrow{\overline{\mathbf{Z}}\mathbf{I}_0}$ and though the origin.

When $\alpha_i(\alpha_i \in \Lambda)$ is perpendicular to vector $\overrightarrow{\overline{\mathbf{Z}}\mathbf{I}_0}$, π_i [the plane though the origin, namely $(\alpha_i, \mathbf{Z}-\mathbf{I}_0) = 0$] and Π [the plane though $\overline{\mathbf{Z}}$ point, namely $(\alpha_i, \mathbf{Z}-\overline{\mathbf{Z}}) = 0$] overlap. Now, clearly $(\alpha_i, \mathbf{I}_0) = h_i$, means $\frac{1}{1+r}w_i = h_i$.

When $\alpha_i(\alpha_i \in \Lambda)$ is not perpendicular to vector $\overrightarrow{\overline{\mathbf{Z}}\mathbf{I}_0}$, d_i (vertical distance from the origin to π_i) and D_i (vertical distance from the origin to Π_i) are different, but we can compare d_i to D_i. On the premise that we do not know $\overline{\mathbf{Z}}$, $\forall \alpha_j \in \Lambda$, according to the relation of α_j and plane Ψ, whether we can compare d_i to D_i or not? If we can compare d_i to D_i, then know big or small of $(\alpha_j, \mathbf{I}_0)$ and $(f(\alpha_j),\mathbf{I}_0)$. When $(\alpha_j, \mathbf{I}_0) < (f(\alpha_j), \mathbf{I}_0)$, then $x_i > 1$; on the contrary, $x_i < 1$. Because $\|\overline{\mathbf{Z}}\| > \|\mathbf{I}_0\|$, if $\forall \alpha_j \in \Lambda$, α_j and $\overline{\mathbf{Z}}$ on same side of plane Ψ, then $D_i > d_i$, that is after transformation price of production is larger than their value. On the contrary, if α_j and $\overline{\mathbf{Z}}$ on different side of plane Ψ, then $D_i > d_i$, that is after transformation price of production is smaller than their value. We must solve problem that how check α_j and $\overline{\mathbf{Z}}$ on which side of plane Ψ. The University of Shimane Nonaka Yasuo professor proposed that with inner product compute angle between vectors, but we do not know how try to find the criterion.

Conclusion

The transformation problem proposed by Bortkiewicz may be ended with the proof of Theorem1 and solution of unit problem in transformation in this paper. But even in static transformation, this does not mean that transformation problem has been completely solved. Example, we have stated that the proposition 4 and 5. Go further into it, the transformation problem is a historic course. Therefore, there exists a dynamic transformation problem.

Concerning static transformation, we must emphasize that F. Seton left an error. F. Seton was the first who used input-output method in the study of transformation problem. This is a great advancement. But it is a pity that he confounded constant capital and variable capital. That is, number of variable is only n in F. Seton. It means labor power is not independent variable. Space being limited, this paper cannot make a complete analysis of Seton's study, but like to point out that this way of thinking is a fatal error too, as transformation is treated as an intermediate link in the transformation between value and price.

Finally, we must make some breakthroughs on value theory, for complete solution of the transformation problem. Example, how rethink the relation of market value and production-price, and others.

Notes

1) Although theoretically value system coexists with production pricing system, and value is regarded as invisible

while the production-price is visible.

2) total the production-price equals total value, and total average profit equals total surplus-value.

3) Refer to Zhang, Z. [2000] and [2001].

4) The first serious error is to identify the ***two major categories*** with ***two departments*** (or two industries). But the two major categories are the result of aggregation of means of production and means of consumption in each department, categorized respectively, and they are not something which really existed. The case of more than two major categories cannot exist. It is entirely impossible to expand two major categories to three major categories as Bortkiewicz did. Even taking one step backward, if the three major categories theory could hold, I am afraid that nobody can further imagine n (more than four) major categories. That is to say, Bortkiewicz's method of expanding major categories cannot be universally established. The second serious error in the past research is that: initiated by Bortkiewicz, attempts to build up transformation formula usually started from equilibrium relation in simple reproduction.

5) By Zhang, Z. [2000] and [2001].

6) The Marx analysis claims invariance for value aggregate and surplus-value. Namely, after transformation total the production-price equals total value, and total average profit equals total surplus-value.

7) For the W. Leontief input/output model, proof of existence of non-negative solution is enough, but for transformation model, only proof of existence of positive solution is significant in economics.

8) *Capital* Vol. I (Reprint. New York: *International Publishers*, 1967), 8

9) *Capital* Vol. I , **53**.

10) For relevant theory change in quantity of value, refer to *Capital* Vol. I , 53-55

11) "But money itself has no price. In order to put in on an equal footing with all other commodities in this respect, we should be obliged to equate it to itself as its own equivalent." *Capital* Vol. I , **95**.

12) As early as in 1942, Sweezy proposed this coefficient, but subsequent researchers mistook this value-price exchange coefficient for a relation of value and the production-price exchange (that is, deviation coefficient between the production-price and value).

13) *Capital* Vol. I , **98**.

14) "the price of production of commodities has been developed as its converted form." Refer to *Capital* Vol. III , **163**.

15) Bortkiewicz made a mistake here-he mixed up deviation rate in the third sector with unit the production-price. M. Desai pointed out this mistake. (Refer to Desai [1979] pp. 77).

16) Although Sweezy summed up inequality between total the production-price and total value said by Bortkiewicz into a subject of calculus unit, the core of his intent was to make currency as the calculus unit for the production-price and for value; and if labor time was used as calculus unit for the production-price and for value, he deemed that they might be equivalent to each other. In short, for Sweezy, the calculus unit of the production-price and of value is the same. In subsequent studies, there is no chaotic concept between the production-price and price. Besides, in his thesis, Sweezy pointed out that if labor time is used as calculus unit for the production-price and for price, N. Moszkowska proposed a very ingenious transformation method. Luckily, the author of this paper found German original version of that book (refer to Reference), and found that this method was not universal, but only further exploration made on the field of definition in the Bortkiewicz model.

17) *Capital* III, pp. 159-60.

18) *Capital* III, p. 165.

19) *Capital* III, p. 167.

20) *Capital* III, p. 163. By M. Morishima [1973] p.73.

21) *Capital* III, p. 164. By M. Morishima [1973] p.73.

22) "But for the buyer the price of production of a specific commodity is its cost-price, and may thus pass as cost-price into the prices of other commodities." *Capital* Vol. III , **164**.

23) We can prove $\|\overline{\mathbf{Z}}\| > \|\mathbf{I}_0\|$. Clearly $\|\mathbf{I}_0\| = \sqrt{n+1}$. As model (14), so

$$\|\overline{\mathbf{Z}}\|^2 = \overline{\mathbf{Z}}'\overline{\mathbf{Z}} = \mathbf{M}'(\mathbf{A}^{-1})'\mathbf{A}^{-1}\mathbf{M} \tag{34}$$

let $\mathbf{I}_1' = (0, \cdots, 0, 1)_{1\times(n-1)}$, $\Omega = \begin{bmatrix} 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \\ 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$, then (34) becomes to

$$\|\overline{\mathbf{Z}}\|^2 = \overline{\mathbf{Z}}'\overline{\mathbf{Z}} = \mathbf{I}_1'(\Omega^{-1}\mathbf{A}'\mathbf{A}\Omega^{-1})^{-1}\mathbf{I}_1 = \frac{(\sum_{i=1}^{n} w_i)^2}{\sum_{i=1}^{n} v_i^2} \tag{35}$$

next we use Cauchy inequality get

$$\frac{(\sum_{i=1}^{n} w_i)^2}{\sum_{i=1}^{n} v_i^2} > n\frac{(\sum_{i=1}^{n} w_i)^2}{(\sum_{i=1}^{n} v_1)^2} > n(1-e+k^{\min})^2 > n+1$$

therefore $\|\overline{\mathbf{Z}}\| > \|\mathbf{I}_0\|$.

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(ZHANG Zhongren)