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The Transformation Problem : Samuelson and Marx Reach the Same Goal by Different Routes

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The transformation problem has been a big barrier for us for more than 100 years and seemed to be solved with no way. However, when I gave the general model of transition which is the first one to satisfy "two propositions of being totally consistent" and thus made a great step forward in 2000, I found that there were actually many methods to solve the problem with mathematics and at least two kinds meaningfully in economics., i.e. the static model and the dynamic model.

Recently, when I was studying interrogation on the model of transition by Samuelson in 1957, I fatherly found that in Samuelson's methods there contained the second way to solve the transformation problem: price of production of Marx is transformed from value, whereas that of Samuelson is "born" by the system of physical objects. That is, with Samuelson, value and price of production have the same roots and closely connected, where "the roots" are the same system of physical objects, and the two "flowers" "born" are respectively the value and the price of production. So, studies by Samuelson disclose that the two systems of value and the price of production could be formed from the same system of physical objects, which could also satisfy the "two propositions of being totally consistent". This is just the best argument for Marx's theory of price of production! From this point of view, we could say that Samuelson and Marx actually reach the same goal by different routes.

Why Samuelson got the opposite results is that he had made a mistake that he shouldn't do. Samuelson let W_π be rate of wage (i.e. value of unit labor), s be rate of surplus value and $\mathbf{I}$ be unit matrix; $\mathbf{a}_0=\left[a_{0j}\right]_{1\times n}$, represents vector of labor worked per unit of commodity; $\mathbf{m}'=\left[m_i\right]_{1\times n}$, represents vector of minimum quantity of commodity needed by labor for living, i.e. wage in kind, therefore, there is the following formula about rate of wage: $\boldsymbol{\pi}\mathbf{m}=\sum\pi_i m_i=W_\pi$. $\boldsymbol{\pi}=\left[\pi_i\right]_{1\times n}$, represents vector of the value per unit of commodity: $\mathbf{a}=\left[a_{ij}\right]_{n\times n}$, represents matrix of coefficients of material consumption. Therefore, Samuelson believes that the formula of value in the 1st volume of capital by Marx could be

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shown as $\boldsymbol{\pi}=W_{\pi}\mathbf{a}_0+\boldsymbol{\pi}\mathbf{a}+sW_{\pi}\mathbf{a}_0$, from the viewpoint of unit commodity; then let $\mathbf{A}_0(0)=\mathbf{a}_0(\mathbf{I}-\mathbf{a})^{-1}$, and collecting the same items of the above term and rearranging, we obtain

$$\left.\begin{aligned}&\boldsymbol{\pi}=W_{\pi}\mathbf{a}_0(\mathbf{I}-\mathbf{a})^{-1}(1+s)=W_{\pi}\mathbf{A}_0(0)(1+s)\\&\boldsymbol{\pi}\mathbf{m}=W_{\pi}\end{aligned}\right\}$$

Samuelson gave the way to solve the model, i.e. the solution for rate of surplus value s is $s=\frac{1}{\mathbf{A}_0(0)\mathbf{m}}-1$.

After quantifying Marx's system of value, Samuelson started to quantify the system of price of production. He let r be average rate of profit, $\mathbf{P}=\left[P_i\right]_{1\times n}$ be vector of price of production, and show the formula of price of production of unit commodity in the 3rd volume of capital by Marx as $\mathbf{P}=(W_p\mathbf{a}_0+\mathbf{P}\mathbf{a})(1+r)$. And then he let $\mathbf{A}_0(r)=\mathbf{a}_0(1+r)[\mathbf{I}-\mathbf{a}(1+r)]^{-1}$ be vector of coefficients of all labor worked under the condition of average profit, collecting the same items of the above term and rearranging, we obtain

$$\left.\begin{aligned}&\mathbf{P}=W_p\mathbf{a}_0(1+r)[\mathbf{I}-\mathbf{a}(1+r)]^{-1}=W_p\mathbf{A}_0(r)\\&\mathbf{P}\mathbf{m}=W_p\end{aligned}\right\}$$

This is just the system of price of production quantified by Samuelson. The way of solution is from $\mathbf{P}=W_p\mathbf{A}_0(r)$ which is a high degree equation with one unknown of r, r is solved and then put into $\mathbf{P}=W_p\mathbf{A}_0(r)$ and thus $\mathbf{P}$ is obtained. But it is quite difficult to calculate because of an unknown of the equation contained in a inverse matrix. This problem was solved by Steadman in 1977. Actually Steadman's method is very simple, which with $\mathbf{P}\mathbf{m}=W_p$ to get of W_p we can get $\mathbf{P}=\mathbf{P}(\mathbf{m}\mathbf{a}_0+\mathbf{a})(1+r)$ and thus $\frac{1}{1+r}$ becomes the feature vector of the matrix of $(\mathbf{m}\mathbf{a}_0+\mathbf{a})$, then the calculation becomes very easy.

Thus, Samuelson believes that the system of value and that of price are separately determined and are two substitutable but unharmonious systems.

At this point, Samuelson made a subjective mistake. The key point here is $\mathbf{m}$ which at the same time decides W_{π} and $W_p(W_{\pi}=\boldsymbol{\pi}\mathbf{m},\ W_p=\mathbf{P}\mathbf{m})$. But the value of $\mathbf{m}$ has infinite possibilities in figure examples (of course in reality the possibility is unique), we could choose $\mathbf{m}^*$ to make the feature value of $(\mathbf{m}\mathbf{a}_0+\mathbf{a})$ is consistent with the definition by Marx, and there are infinite $\mathbf{m}^*$ satisfying this kind of condition. As far as the system of price of production is concerned, we still need another constraint condition $\mathbf{P}\mathbf{E}=\boldsymbol{\pi}\mathbf{E}$ (the total amount of price of production equals the total amount of value), by which the chosen $\mathbf{m}^*$ could assure the total amount of average profit equals the total value of surplus value. Thus, we get the same system of matrix of physical objects $\mathbf{a}$, and the system of value and that of price of production produced by $\mathbf{a}$, $\mathbf{a}_0$, $\mathbf{m}^*$ are as follows

$$\left.\begin{array}{l} \boldsymbol{\pi} = W_\pi \mathbf{A}_0(0)(1+s) \\ \boldsymbol{\pi} m = W_\pi \\ \mathbf{m} = \mathbf{m}^* \end{array}\right\}$$

and

$$\left.\begin{array}{l} \mathbf{P} = W_p \mathbf{A}_0(r) \\ \mathbf{Pm}^* = W_p \\ \mathbf{PE} = \boldsymbol{\pi}\mathbf{E} \end{array}\right\}$$

By this we can see that price of production could also be “born” by the system of physical objects and at the same time satisfy “two propositions of being totally consistent”. This is another kind of understanding about the price of production, or another kind of theory of the price of production, but is surprisingly consistent with Marx’s theory of the price of production.

Then, how comes Marx’s price of production? It is transformed from value. Here we especially emphasize the words of “transform” of which, Bortkiewicz’s comprehension comparatively accords with Marx’s thoughts. If Bortkiewicz hasn’t fallen into the trap of the wrong 3 departments and he could see the table of input and output, probably he has solved the transformation problem.

To illustrate the problem, we just employ Samuelson’s sign, but add some others to that. First, let $\frac{P_i}{\pi_i} = x_i$ show the deviation rate of price from value and also let $y = \frac{W_p}{W_\pi}$ represent the deviation rate of the rate of wage and put them into the system of price of production to get a system of equations. Here we treat average profit r as an unknown. But now there are all together $n+2$ unknowns $x_1, \cdots, x_n, y, r$, whereas there are only n equations, i.e. two degree of freedom. So, to get a unique solution needs addition of two equations. We thus turn “two propositions of being totally consistent” as the condition of constraint into two equations and put them into the system, the mathematical model of transforming the system of value $(\boldsymbol{\pi} = \boldsymbol{\pi}\mathbf{a} + (1+s) W_\pi \mathbf{a}_0)$ into the system of price of production $(\mathbf{P} = (1+r)(\mathbf{Pa} + W_p \mathbf{a}_0))$ is then obtained and shown as follows[1)]

$$\begin{cases} \mathbf{X}\boldsymbol{\pi} = (yW_\pi \mathbf{a}_0 + \mathbf{X}\boldsymbol{\pi}\mathbf{a})(1+r) \\ (yW_\pi \mathbf{a}_0 + \mathbf{X}\boldsymbol{\pi}\mathbf{a})\mathbf{E} = (W_\pi \mathbf{a}_0 + \boldsymbol{\pi}\mathbf{a})\mathbf{E} \qquad (1+r)\sum_{i=1}^{n} a_{ij} < \pi_j \quad (j=1,2,\cdots,n) \\ \mathbf{X}\boldsymbol{\pi}' = \boldsymbol{\pi}\mathbf{E} \end{cases}$$

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1) The condition $(1+r)\sum_{i=1}^{n} a_{ij} < \pi_j$, by Huan Zhongdan and Zhang Zhongren, A Necessary and Sufficient Condition of Positive Solutions to the BSZ Transformation Model, Shimane Journal of Policy Studies, Vol.9, March 2005.

The model satisfies "two propositions of being totally consistent" and has the unique positive solution.

The model of "born" price of production of Samuelson is complement with Marx's transforming problem and constitutes another profile of the theory of price of production.

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